

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. SECOND SEMESTER EXAMINATION, MAY 2017

FIRST YEAR [BATCH 2016-18]

APPLIED CHEMISTRY

Date : 20/05/2017

Time : 11 am – 1 pm

Paper : VII [Engineering Mathematics-I]

Full Marks : 50

[Use a separate Answer Book for each group]

Group - A

Answer **any six** questions from **Question Nos. 1 to 9** :

[6 X 5]

1. Find the adjoint and hence inverse of the matrix:

3+2

$$\begin{pmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

2. Obtain the rank of the matrix:

5

$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{pmatrix}$$

3. Show that the equations:

5

$$x - 3y - 8z = -10$$

$$3x + y - 4z = 0$$

$$2x + 5y + 6z = 13$$

are consistent and solve them.

4. Find the characteristic polynomial of the following matrix:

5

$$A = \begin{bmatrix} 4 & 3 & 0 & -1 \\ 1 & 2 & -5 & 4 \\ 3 & 0 & 1 & -2 \\ -2 & 1 & 0 & -3 \end{bmatrix}$$

5. Find A^{-1} by using elementary row operations; where $A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 4 \\ 3 & 3 & 7 \end{pmatrix}$.

5

6. Prove that $u = x^2 - y^2$, $v = -y/(x^2 + y^2)$; both u and v satisfy Laplace's equation but $u + vi$ is not an analytic function of z .

5

7. Evaluate $\oint_c \frac{3z^2 + z}{z^2 - 1} dz$, where c is the circle $|z - 1| = 1$.

5

8. Find Laurent series about the indicated singularity for the following functions:
- (i) $\frac{e^{2z}}{(z-1)^3}$; $z=1$. 2½
- (ii) $(z-3)\sin\frac{1}{z+2}$; $z=-2$. 2½
9. Find the residue of $f(z) = \frac{\sin z}{z^7}$ at $z=0$. 5

Group - B

Answer **any four** questions from **Question Nos. 10 to 15** : [4 X 5]

10. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by, $f(x) = |x-1|^2 + |x-2|$. Is f continuous over $\mathbb{R}$? If not find the point of its discontinuity with proper explanation. 5
11. (i) Find the extreme values of the function $f(x) = x^5 - 5x^4 + 5x^3 + 12$.
(ii) Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{1/e}$. 5
12. State and prove Euler's theorem on homogeneous function in case of three variables. 5
13. Determine the radius of convergence of the power series (i) $\sum_{n=1}^{\infty} \frac{n^n}{n} x^n$, (ii) $1 + \sum_{n=2}^{\infty} \frac{(\lfloor n \rfloor)^2}{\lfloor 2n \rfloor} x^n$. 5
14. a) Using μ -test show that $\int_1^{\infty} \frac{(x-1)\sqrt{x}}{1+x+x^3+\sin x} dx$ is convergent.
b) Prove that $\int_{-1}^1 \frac{dx}{x^3}$ exists in Cauchy's principal value sense although the integral does not exist. 5
15. Define $\Gamma(n)$, the gamma function & $\beta(m,n)$, the Beta function. For the gamma function, show that (i) $\Gamma(n+1) = n\Gamma(n)$ & (ii) $\Gamma(n+1) = \lfloor n \rfloor$ if n is a positive integer. 5

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